# Uncertainty Quantification in Deep Learning



Jiyoon Lee 2022 . 01 . 28



발표자 소개



#### ❖ 이지윤 (Jiyoon Lee)

- Data Mining & Quality Analytics Lab
- Ph.D. Candidates (2018.03 ~ Present)

#### Research Interest

- Explainable neural network using Attention mechanism & Bayesian neural network
- Graph-based semi-supervised learning using Label propagation

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- E-mail: jiyoonlee@korea.ac.kr





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- Background
- Uncertainty

#### 2. Bayesian-based Approach

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#### 3. Ensemble-based Approach

- Simple and Scalable Deep Ensembles

#### 4. GP-based Approach

- Spectral-normalized Neural Gaussian Process

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- Uncertainty Baselines

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Background

## **Expectation**

Training dataset

Dogs



**Driving** 

Background

## **Expectation**

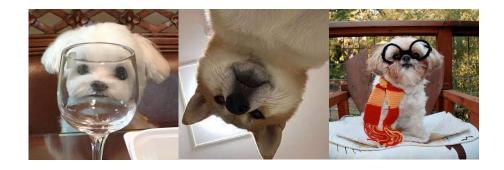
Training dataset

Dogs



Reality

Testing in reality







#### **Background**



#### [이슈톡] '자율주행'이라더니...테슬라, 전복 트럭도 못 피하고 정면 충돌

입력 2020-06-03 06:51 | 수정 2020-06-03 09:22











#### "테슬라 사고는 역광 때문"…는 · 비 등 '악천후', 자율주행 난관으로 떠올라

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Background

Scene Understanding

**Facial Detection** 



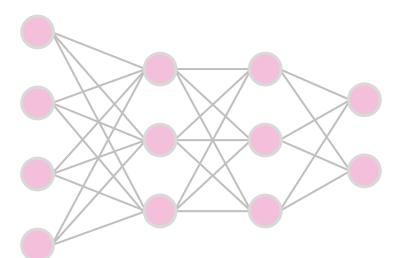
얼마나 확신하는지에 대한 지표 "Uncertainty"
ACCESS 불확실성 FORMATION CYBER SECURITY
\*\*Uncertainty"를 예측 확률로 정량화 해보자!
\*\*OMPUTER\*\*

**Uncertainty** 

#### Standard Neural Networks

• SoftMax를 통해 logit값을 확률 값으로 변환함으로써 예측 확률이 도출





#### **Certain!**

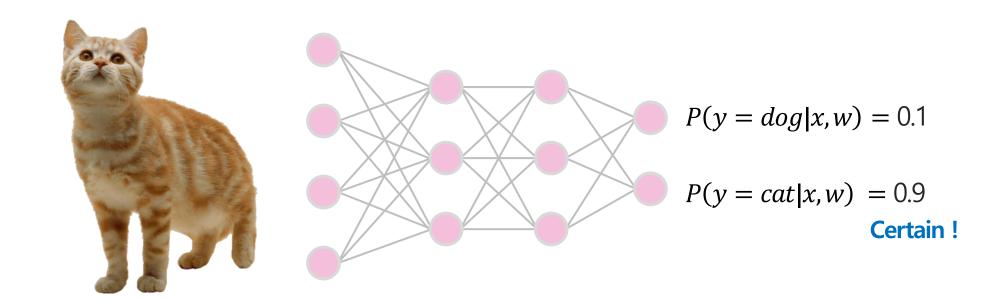
$$P(y = dog|x, w) = 0.9$$

$$P(y = cat | x, w) = 0.1$$

#### **Uncertainty**

#### Standard Neural Networks

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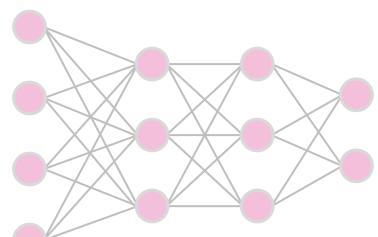


**Uncertainty** 

#### Standard Neural Networks

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#### **Uncertain ???**

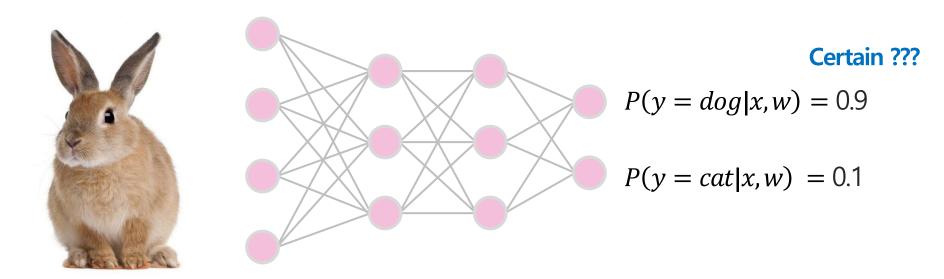
$$P(y = dog|x, w) = 0.5$$

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Uncertainty

#### **❖ Standard Neural Networks**

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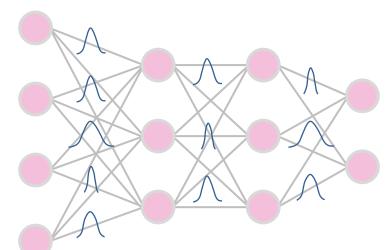
"Uncertainty"를 예측 확률로 정량화 하는 것은 한계가 있음!

#### **Uncertainty**

#### Bayesian Neural Networks

- Parameter w 에 분포를 가정하여, 예측 값을 분포로 추정할 수 있음
- 도출된 예측값의 분산정보를 활용하여 불확실성 정량화가 가능





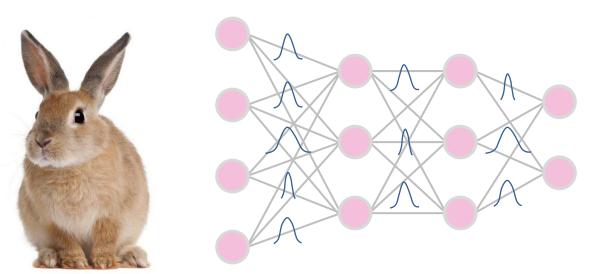
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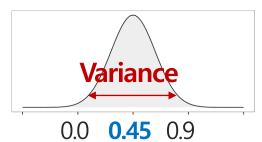
Uncertainty

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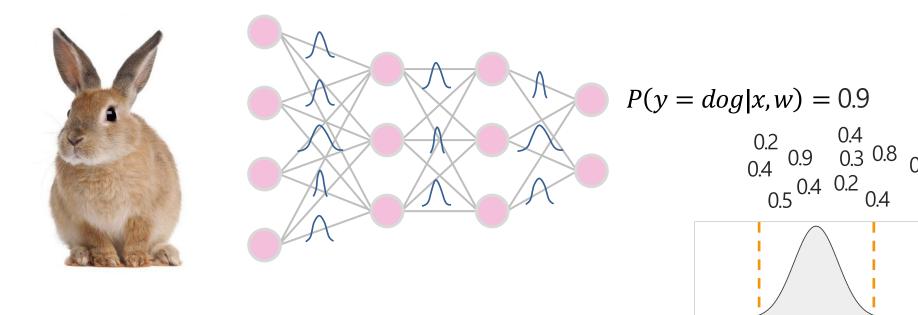


"Uncertainty"를 예측 확률의 분산으로 정량화 하고자 함

**Uncertainty** 

#### Bayesian Neural Networks

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0.0 **0.45** 0.9

**Confidence Interval** 

# **Uncertainty Quantification**



# Uncertainty

예측에 대한 불확실성

# Quantification

# 잘 정량화하자

분산정보를 어떻게 도출할지 분산정보를 어떻게 요약할지

#### **Bayesian-based Approach**

Dropout as a Bayesian Approximation:

Representing Model Uncertainty in Deep Learning

Voit Gal

Zushie Chadramani

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#### **Ensemble-based Approach**



#### **GP-based Approach**



# Uncertainty

# Quantification

# 예측에 대한 불확실성

잘 정량화하자

모델이 불확실한 경우 데이터가 불확실한 경우

#### **Bayesian-based Approach**

# Dropout as a Bayesian Approximation: Representing Model Uncertainty in Deep Learning Visit State of Computer Vision? Deep learning of Combusting Vision? Deep learning of Combusting Vision State of Computer Vision? Deep learning of Combusting Vision State of Computer Vision? The property of the State of Compute

#### **Ensemble-based Approach**



#### **GP-based Approach**





#### **Types of Uncertainty**

#### Epistemic uncertainty (model uncertainty)

- 모델이 데이터에 대해 얼마나 적합하게 구축되었는지에 대해 모르는 정도
- 데이터의 어떤 특징을 학습하는지에 대해 모르는 정도
- 더 많은 데이터가 학습된다면 줄일 수 있음, reducible uncertainty

#### Aleatoric uncertainty (data uncertainty)

- 데이터에 내재된 노이즈로 인해 이해하지 못하는 정도 (e.g. measurement noise, randomness inherent)
- 더 많은 데이터가 학습되더라도 줄일 수 없음, irreducible uncertainty
- 측정 정밀도를 높이면 줄일 수 있음



"테슬라 사고는 역광 때문"…눈 · 비 등 '악천후', 자율주행 난관으로 떠올라

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#### Why Epistemic uncertainty?

- Epistemic uncertainty는 학습데이터가 부족하여 학습되지 않은 상태를 식별할 수 있기 때문에 중요
- 높은 불확실성은 모델은 추가적인 학습이 필요할 가능성이 높다는 의미로, 안전이 중요한 문제상황에서 높게 발생하는 경우
   모델을 신뢰할 수 없음

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#### Why Aleatoric uncertainty?

- Aleatoric uncertainty는 실제 상황에서와 같이 일부 데이터의 노이즈가 높게 존재하는 경우 중요
- 노이즈가 큰 데이터에 대해 학습과정에서 제약을 부여할 수 있으므로, 예측 성능 안정화 과정에 기여



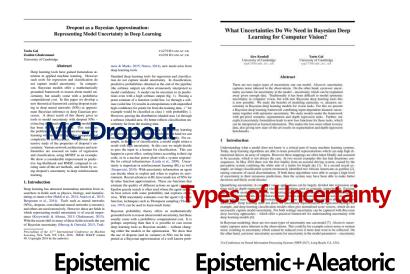
# Uncertainty

# Quantification

예측에 대한 불확실성

잘 정량화하자

#### **Bayesian-based Approach**



**Ensemble-based Approach** 



**GP-based Approach** 



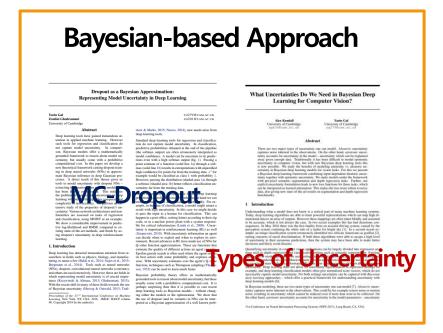
Out-of-distribution (calibrated probability)

# Uncertainty

# Quantification

예측에 대한 불확실성

잘 정량화하자



#### **Ensemble-based Approach**



#### **GP-based Approach**





#### **Dropout as Bayesian Approximation**

#### ❖ ICML 2016 (22년 1월 기준 4670건 인용)

#### Dropout as a Bayesian Approximation: Representing Model Uncertainty in Deep Learning

Yarin Gal Zoubin Ghahramani University of Cambridge

#### Abstract

Deep learning tools have gained tremendous attention in applied machine learning. However such tools for regression and classification do not capture model uncertainty. In comparison, Bayesian models offer a mathematically grounded framework to reason about model uncertainty, but usually come with a prohibitive computational cost. In this paper we develop a new theoretical framework casting dropout training in deep neural networks (NNs) as approximate Bayesian inference in deep Gaussian processes. A direct result of this theory gives us tools to model uncertainty with dropout NNs extracting information from existing models that has been thrown away so far. This mitigates the problem of representing uncertainty in deep learning without sacrificing either computational complexity or test accuracy. We perform an extensive study of the properties of dropout's uncertainty. Various network architectures and nonlinearities are assessed on tasks of regression and classification, using MNIST as an example. We show a considerable improvement in predictive log-likelihood and RMSE compared to existing state-of-the-art methods, and finish by using dropout's uncertainty in deep reinforcement

#### 1. Introduction

Deep learning has attracted tremendous attention from ressearchers in fields such as physics, biology, and manufacturing, to name a few (Baldi et al., 2014; Anjos et al., 2015; Bergmann et al., 2014). Tools such as neural networks (NNS), dropout, convolutional neural networks (convnets), and others are used extensively. However, these are fields in which representing model uncertainty is of crucial importance (Krzywinski & Altman, 2013; Ghahramani, 2015).

Proceedings of the 33<sup>rd</sup> International Conference on Machine Learning, New York, NY, USA, 2016. JMLR: W&CP volume 48. Copyright 2016 by the author(s). YG279@CAM.AC.UK ZG201@CAM.AC.UK

With the recent shift in many of these fields towards the use of Bayesian uncertainty (Herzog & Ostwald, 2013; Trafimow & Marks, 2015; Nuzzo, 2014), new needs arise from deep learning tools.

Standard deep learning tools for regression and classification do not capture model uncertainty. In classification, predictive probabilities obtained at the end of the pipeline (the softmax output) are often erroneously interpreted as model confidence. A model can be uncertain in its predictions even with a high softmax output (fig. 1). Passing a point estimate of a function (solid line 1a) through a softmax (solid line 1b) results in extrapolations with unjustified high confidence for points far from the training data. x\* for example would be classified as class 1 with probability 1. However, passing the distribution (shaded area 1a) through a softmax (shaded area 1b) better reflects classification uncertainty far from the training data.

Model uncertainty is indispensable for the deep learning practitioner as well. With model confidence at hand we can treat uncertain inputs and special cases explicitly. For example, in the case of classification, a model might return a result with high uncertainty. In this case we might decide to pass the input to a human for classification. This can happen in a post office, sorting letters according to their zip code, or in a nuclear power plant with a system responsible for critical infrastructure (Linda et al., 2009). Uncertainty is important in reinforcement learning (RL) as well (Szepesvári, 2010). With uncertainty information an agent can decide when to exploit and when to explore its environment. Recent advances in RL have made use of NNs for Q-value function approximation. These are functions that estimate the quality of different actions an agent can take. Epsilon greedy search is often used where the agent selects its best action with some probability and explores otherwise. With uncertainty estimates over the agent's Q-value function, techniques such as Thompson sampling (Thompson, 1933) can be used to learn much faster.

Bayesian probability theory offers us mathematically grounded tools to reason about model uncertainty, but these usually come with a prohibitive computational cost. It is perhaps surprising then that it is possible to east recent



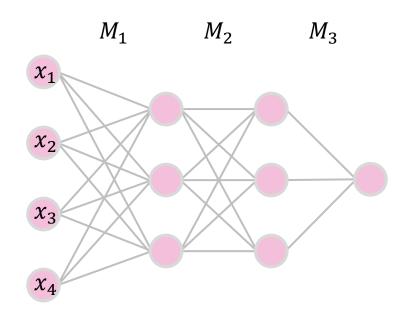
Yarin Gal
Associate Professor, <u>University of Oxford</u>
cs.ox.ac.uk의 이메일 확인됨 - <u>홈페이지</u>
Machine Learning Artificial Intelligence Probability Theory Statistics

☑ 팔로우

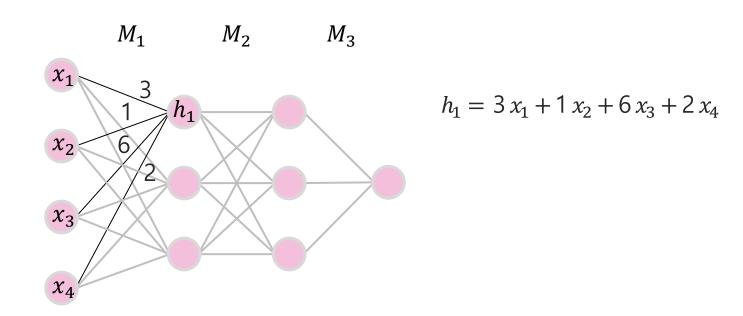
제목	인용	연도
<u>Dropout as a Bayesian approximation: Representing model uncertainty in deep learning</u> Y Gal, Z Ghahramani Proceedings of the 33rd International Conference on Machine Learning (ICML-16)	4721	2015
What uncertainties do we need in Bayesian deep learning for computer vision?  A Kendall, Y Gal  Advances in neural information processing systems, 5574-5584	2403	2017
A theoretically grounded application of dropout in recurrent neural networks Y Gal, Z Ghahramani Advances in neural information processing systems 29, 1019-1027	1499	2016
Multi-task learning using uncertainty to weigh losses for scene geometry and semantics A Kendall, Y Gal, R Cipolla Proceedings of the IEEE Conference on Computer Vision and Pattern	1409	2018
Uncertainty in Deep Learning Y Gal University of Cambridge	1211	2016
Deep Bayesian Active Learning with Image Data Y Gal, R Islam, Z Ghahramani International Conference on Machine Learning (ICML), 1183-1192	819	2017
Bayesian Convolutional Neural Networks with Bernoulli Approximate Variational Inference Y Gal, Z Ghahramani 4th International Conference on Learning Representations (ICLR) workshop track	545	2015



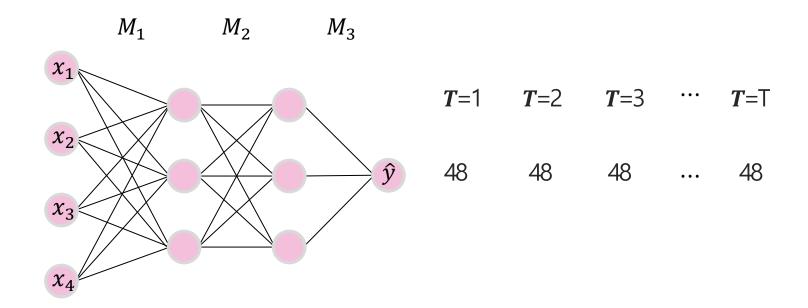
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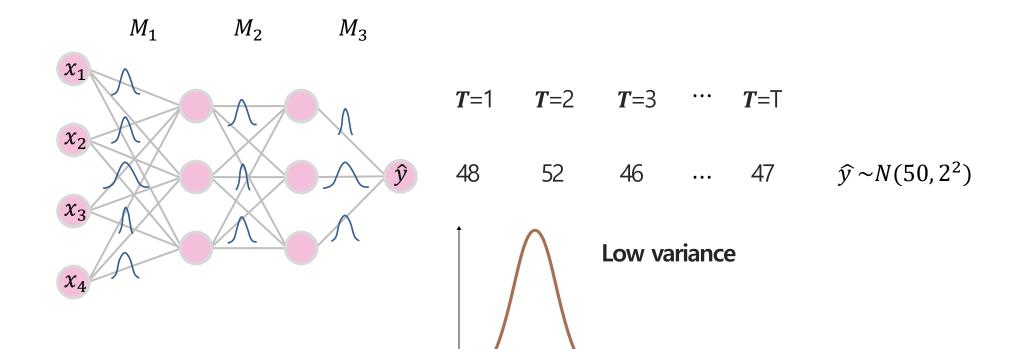
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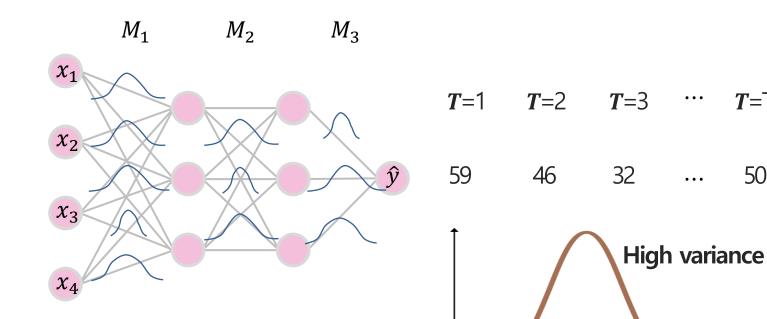


- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
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Frequentist way & Bayesian way

- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
  - 학습 후, 동일한 입력 값에 대해 서로 다른 예측 값이 도출



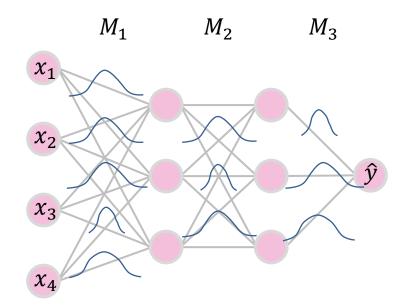
어떻게 파라미터 분포를 추정할까?

*T*=T

 $\hat{y} \sim N(50, 10^2)$ 

**Dropout as Bayesian Approximation** 

- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
  - MC dropout과 weight에 L2 regularization을 적용시키는 것으로 동일한 효과임을 증명



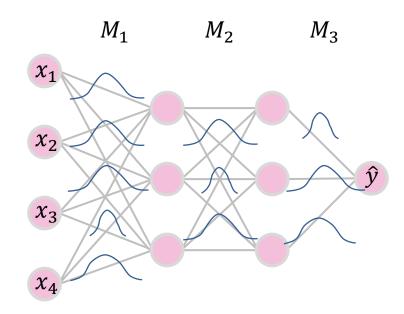
Posterior
$$p(W|X,Y) = \frac{p(Y|X,W)p(w)}{p(Y|X)}$$
Evidence

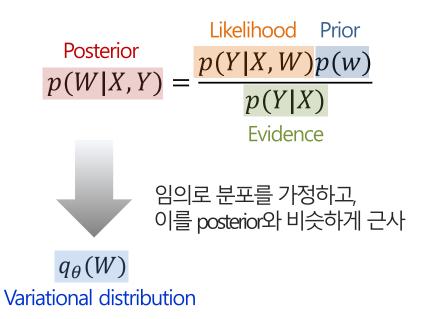
$$\frac{p(Y|X)}{p(Y|X,W)} = \int p(Y|X,W)p(W)dW$$
Evidence

This integration is not computable in general

**Dropout as Bayesian Approximation** 

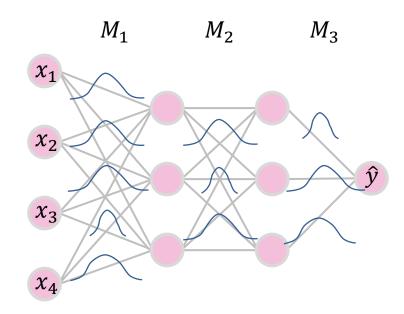
- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
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**Dropout as Bayesian Approximation** 

- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
  - MC dropout과 weight에 L2 regularization을 적용시키는 것으로 동일한 효과임을 증명



#### Variational inference

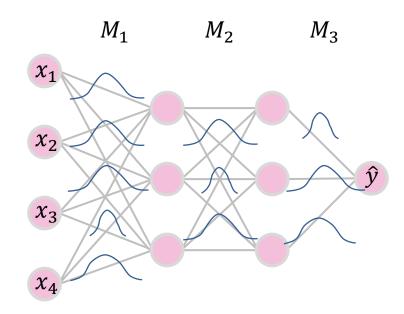
Kullback-Leibler Divergence (두 확률분포의 차이를 계산)

$$q_{\theta}(W)^* = \underset{q \in Q}{argmin} \ KL(q_{\theta}(W)||p(W|X,Y))$$

Variational distribution

**Dropout as Bayesian Approximation** 

- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
  - MC dropout과 weight에 L2 regularization을 적용시키는 것으로 동일한 효과임을 증명



# MC dropout with L2 regularization Variational inference

Kullback-Leibler Divergence (두 확률분포의 차이를 계산)

$$q_{\theta}(W)^* = \underset{q \in Q}{argmin} \ KL(q_{\theta}(W)||p(W|X,Y))$$

**Dropout as Bayesian Approximation** 

❖ Loss function 정의 (Appendix 참고)

Minimize  $KL(q_{\theta}(W)||p(W|X,Y))$ 

- = Maximize ELBO
- $= Minimize \sum_{i=1}^{N} \int q_{\theta}(W) ln \left( p(y_i | f^w(x_i)) \right) dw + KL(q_{\theta}(W) | | p(W))$
- $= Minimize \frac{N}{M} \sum_{i \in S} ln(p(y_i|f^{g(\theta,\epsilon)}(x_i))) + KL(q_{\theta}(W)||p(W))$
- $= Minimize \frac{1}{M} \sum_{i \in S} ln(p(y_i|f^{g(\theta,\hat{\epsilon})}(x_i)) + \lambda_1 ||M_1||^2 + \lambda_2 ||M_2||^2 + \lambda_3 ||b||^2$

 $g(\theta,\hat{\epsilon})=w_{l,i}$ 

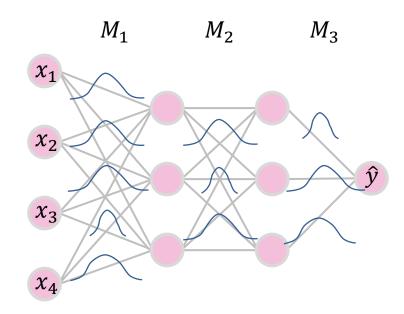
Regression: MSE

Classification: Softmax cross entropy



**Dropout as Bayesian Approximation** 

- ❖ Bayesian : Bayesian Deep learning / Stochastic Deep Learning
  - MC dropout과 weight에 L2 regularization을 적용시키는 것으로 동일한 효과임을 증명



#### MC dropout with L2 regularization

#### Variational inference

Kullback-Leibler Divergence (두 확률분포의 차이를 계산)

$$q_{\theta}(W)^* = \underset{q \in Q}{argmin} \ KL(q_{\theta}(W)||p(W|X,Y))$$

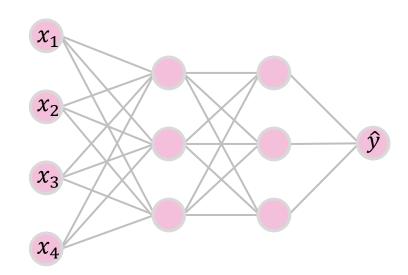
Variational distribution

**Dropout as Bayesian Approximation** 

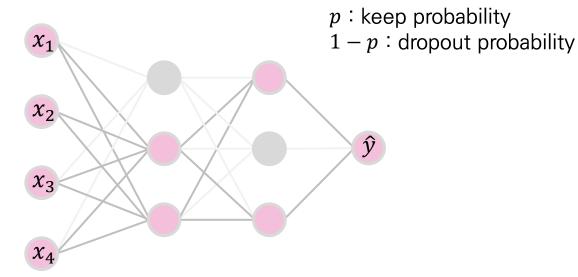
#### Dropout

- 학습데이터에 대한 오버피팅을 방지하기 위한 다양한 정규화 방법론들이 있음
- Dropout은 대표적인 모델 정규화 방법으로 배치마다 무작위로 노드 연결을 끊음
- 이때, dropout을 적용할 비율(1-p)은 사용자가 정의

#### **Standard Neural Network**



#### After applying dropout

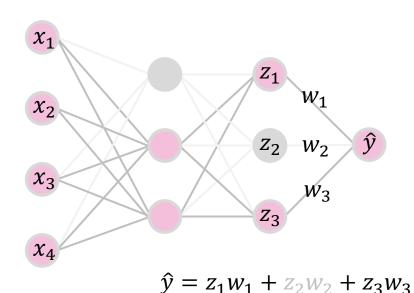


**Dropout as Bayesian Approximation** 

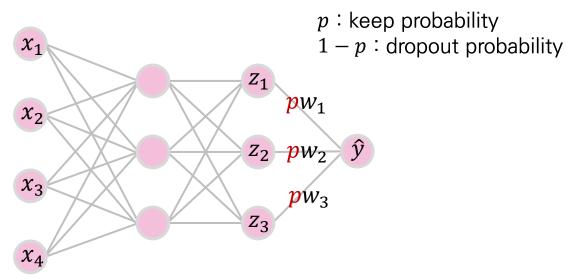
#### Dropout

- Dropout은 추론(inference / test)단계에서는 파라미터를 고정적으로 모델링 (deterministic)
- 고정적인 파라미터에 가중치 p를 곱하여 최종적인 예측 수행  $\rightarrow$  결과는 고정적

#### **Training Phase**



#### **Testing Phase**



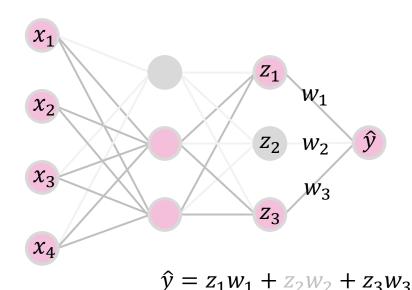
$$\hat{y} = pz_1w_1 + pz_2w_2 + pz_3w_3$$

**Dropout as Bayesian Approximation** 

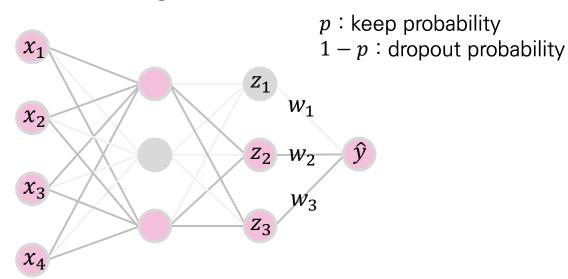
#### Monte Carlo Dropout (MC dropout)

- MC dropout은 추론(inference / test)단계에서도 dropout 적용하여 파라미터 확률적으로 모델링 (Stochastic)
- 매번 추론할 때(stochastic forward pass, **T**)마다 상이한 예측이 수행 → 결과는 확률적

### **Training Phase**



### Testing Phase (T=1)



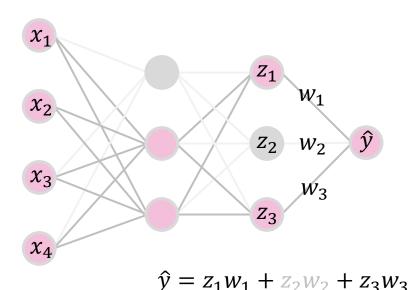
$$\widehat{y_1} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

**Dropout as Bayesian Approximation** 

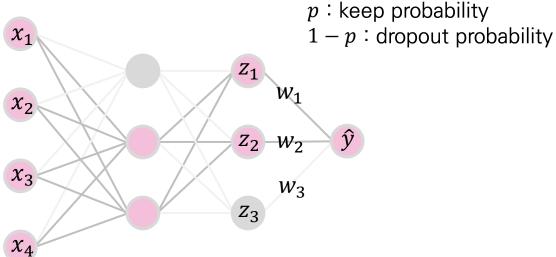
#### Monte Carlo Dropout (MC dropout)

- MC dropout은 추론(inference / test)단계에서도 dropout 적용하여 파라미터 확률적으로 모델링 (Stochastic)
- 매번 추론할 때(stochastic forward pass, **T**)마다 상이한 예측이 수행 → 결과는 확률적

### **Training Phase**



# Testing Phase (T=2)



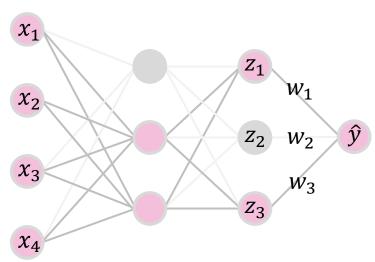
$$\widehat{y_2} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

**Dropout as Bayesian Approximation** 

#### Monte Carlo Dropout (MC dropout)

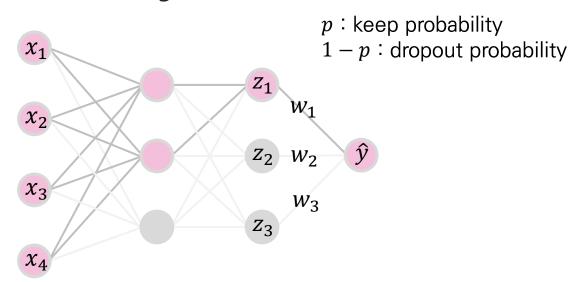
- MC dropout은 추론(inference / test)단계에서도 dropout 적용하여 파라미터 확률적으로 모델링 (Stochastic)
- 매번 추론할 때(stochastic forward pass, **T**)마다 상이한 예측이 수행 → 결과는 확률적

### **Training Phase**



$$\hat{y} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

#### Testing Phase (T=3)



$$\widehat{y_3} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

**Dropout as Bayesian Approximation** 

#### Monte Carlo Dropout (MC dropout)

- MC dropout은 추론(inference / test)단계에서도 dropout 적용하여 파라미터 확률적으로 모델링 (Stochastic)
- 매번 추론할 때(stochastic forward pass, **T**)마다 상이한 예측이 수행 → 결과는 확률적

### **Testing Phase**

$$\widehat{y_1} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

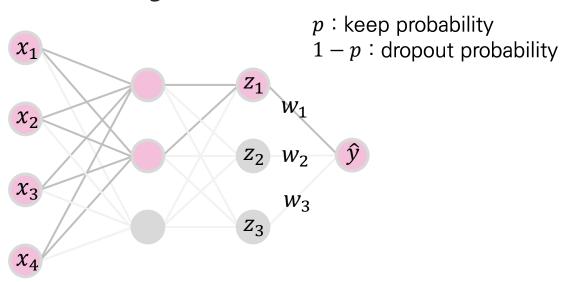
$$\widehat{y_2} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

$$\widehat{y_3} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

$$\vdots$$

$$\widehat{y_T} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

#### Testing Phase (T=3)



$$\widehat{y_3} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

**Dropout as Bayesian Approximation** 

#### Monte Carlo Dropout (MC dropout)

- MC dropout은 추론(inference / test)단계에서도 dropout 적용하여 파라미터 확률적으로 모델링 (Stochastic)
- 매번 추론할 때(stochastic forward pass, **T**)마다 상이한 예측이 수행 → 결과는 확률적

### **Testing Phase**

$$\widehat{y_1} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

$$\widehat{y_2} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

$$\widehat{y_3} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

$$\vdots$$

$$\widehat{y_T} = z_1 w_1 + z_2 w_2 + z_3 w_3$$

After T stochastic forward passes

$$E(y) \approx \frac{1}{T} \sum_{t=1}^{T} \hat{y}_t$$
 Prediction

$$Var(y) \approx \tau^{-1} I_{D} + \frac{1}{T} \sum_{t=1}^{T} \hat{y}_{t}^{T} \hat{y}_{t} - E(y)^{T} E(y)$$

$$\tau = \frac{pl^2}{2N\lambda}$$
 p:probability of units not being dropped

**Epistemic uncertainty** 

**Bayesian Neural Networks for Computer Vision** 

#### ❖ NeurlPS 2017 (22년 1월 기준 2403건 인용)

- Uncertainty 정량화를 세분화함 (Epistemic uncertainty, Aleatoric uncertainty)
- Computer vision tasks에 적용

#### What Uncertainties Do We Need in Bayesian Deep Learning for Computer Vision?

Alex Kendall University of Cambridge Yarin Gal University of Cambridge yg279@cam.ac.uk

#### Abstract

There are two major types of uncertainty one can model. Aleatoric uncertainty captures noise inherent in the observations. On the other hand, epitemic uncertainty accounts for uncertainty in the model – uncertainty accounts for uncertainty in the model – uncertainty which can be explained away given enough data. Traditionally it has been difficult to model epistemic uncertainty in computer vision, but with new Bayesian deep learning tools this is now possible. We study the benefits of modeling epistemic vs. acatoric uncertainty in Bayesian deep learning models for vision tasks. For this we present a Bayesian deep learning framework combining input-dependent aleatoric uncertainty together with epistemic uncertainty. We study models under the framework with per-pixel semantic segmentation and depth regression tasks. Further, our explicit uncertainty formulation leads to new loss functions for these tasks, which can be interpreted as learned attenuation. This makes the loss more robust to noisy data, also giving new state-of-the-art results on segmentation and depth regression benchmarks.

#### 1 Introduction

Understanding what a model does not know is a critical part of many machine learning systems. Today, deep learning algorithms are able to learn powerful representations which can map high dimensional data to an array of outputs. However these mappings are often taken blindly and assumed to be accurate, which is not always the case. In two recent examples this has had disastrous consequences. In May 2016 there was the first fatality from an assisted driving system, caused by the preception system confusing the white side of a trailer for bright sky [1]. In a second recent example, an image classification system erroneously identified two African Americans as gorillas [2], raising concerns of racial discrimination. If both these algorithms were able to assign a high level of uncertainty to their erroneous predictions, then the system may have been able to make better decisions and likely avoid disaster.

Quantifying uncertainty in computer vision applications can be largely divided into regression settings such as depth regression, and classification settings such as depth regression, and classification settings unclassification consolidation and control of the control of the

In Bayesian modeling, there are two main types of uncertainty one can model [7]. Aleatoric uncertainty captures noise inherent in the observations. This could be for example sensor noise or motion noise, resulting in uncertainty which cannot be reduced even if more data were to be collected. On the other hand, epistemic uncertainty accounts for uncertainty in the model parameters – uncertainty

31st Conference on Neural Information Processing Systems (NIPS 2017), Long Beach, CA, USA.



Alex Kend	Alex Kendall										
University of Ca cam.ac.uk의 이미	<u>mbridge</u> 메일 확인됨 - <u>홈페</u> 0	기지									
Deep Learning	Computer Vision	Robotics	Control								

☑ 팔로우

#### https://alexgkendall.com/

제목		인용	연도
V Badrinarayanan, A	convolutional encoder-decoder architecture for scene segmentation Kendall, R Cipolla pattern analysis and machine intelligence	10632	2017
A Kendall, Y Gal	es Do We Need in Bayesian Deep Learning for Computer Vision?	2392	2017
A Kendall, M Grimes	olutional Network for Real-Time 6-DOF Camera Relocalization R Cipolla EEE International Conference on Computer Vision	1552	2015
A Kendall, Y Gal, R (	g using uncertainty to weigh losses for scene geometry and semantics ipolia IEE Conf. on Computer Vision and Pattern Recognition	1406	2018
for Scene Unders A Kendall, V Badrina		881	2017
A Kendall, H Martiros	ning of Geometry and Context for Deep Stereo Regression yan, S Dasgupta, P Henry, R Kennedy, A Bachrach, EEE International Conference on Computer Vision	807 *	2017
A Kendall, R Cipolla	unctions for camera pose regression with deep learning	529	2017



## Introduction

#### **Types of Uncertainty**

#### Epistemic uncertainty (model uncertainty)

- 모델이 데이터에 대해 얼마나 적합하게 구축되었는지에 대해 모르는 정도
- 데이터의 어떤 특징을 학습하는지에 대해 모르는 정도
- 더 많은 데이터가 학습된다면 줄일 수 있음, reducible uncertainty

#### Aleatoric uncertainty (data uncertainty)

- 데이터에 내재된 노이즈로 인해 이해하지 못하는 정도 (e.g. measurement noise, randomness inherent)
- 더 많은 데이터가 학습되더라도 줄일 수 없음, irreducible uncertainty
- 측정 정밀도를 높이면 줄일 수 있음



"테슬라 사고는 역광 때문"…눈 · 비 등 '악천후', 자율주행 난관으로 떠올라

미 ABC 방송의 서부지역 네트워크인 KGO-TV는 이번 테슬라 운전자 월터 후앙의 사망 사고가 지난해 9월 발생한 테슬라의 자동주행 차량 사고와 비슷하다고 최근 보도했다. 지난달 테슬라의 사고는 오전 역광이 내리쬐는 상황에서 차량이 중앙 분리대를 들이받아 발생했는데, 6개월 전 사고도 오전 역광으로 눈부신 상황이었다는 것이다. 이에 앞서 테슬라는 지난 2016년 발생한 트레일러 충돌 사고에 대해 "자동주행 차량이 역광 탓에 흰색 트레일러를 하늘로 오인해 충돌사고를 냈다"고 사고 원인을 밝힌 바 있다.



구조차량이 촬영한 지난해 9월 테슬라 자동주행(오토파일럿 모드) 차량의 중앙분리대 충돌사고 현장. 지난달 사망 사고처럼 오전 역광이 내리쬐는 상황에서 발생했다. 2016년 발생한 트레일러 충돌사고도 역광이 원인이었다.[미 AB( 바소 까지)

## Introduction

**Types of Uncertainty** 

#### Aleatoric uncertainty (data uncertainty)

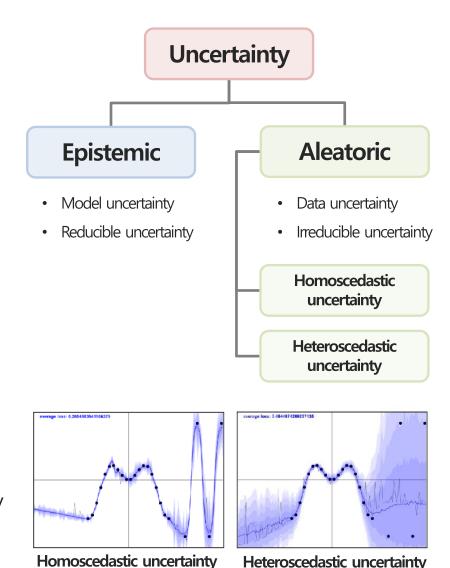
- 데이터에 내재된 노이즈로 인해 이해하지 못하는 정도 (e.g. measurement noise, randomness inherent)
- 더 많은 데이터가 학습되더라도 줄일 수 없음, irreducible uncertainty
- 측정 정밀도를 높이면 줄일 수 있음

#### Homoscedastic uncertainty

• 서로 다른 입력 값에 대해서 동일한 상수값을 지님

#### Heteroscedastic uncertainty

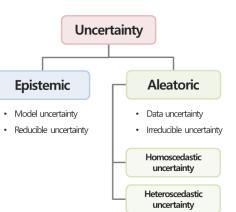
- 서로 다른 입력 값에 대해서 다른 값을 지님, input-dependent uncertainty
- 적절하게 정량화 되는 경우 outlier의 지표로 활용할 수 있음



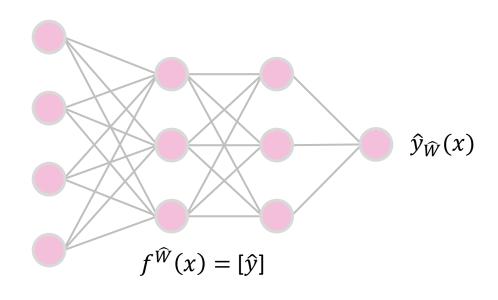
**Bayesian Neural Networks for Computer Vision** 

#### Density Network Architecture

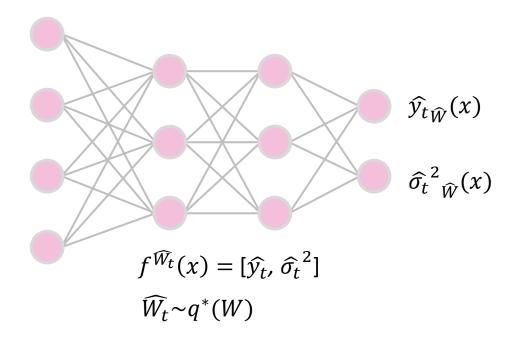
• MC dropout구조와 유사하며, aleatoric uncertainty를 추정하기 위한 output node가 추가된 형태



#### **Standard Neural Network**



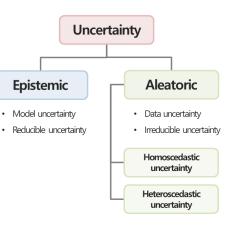
#### **Density Network**

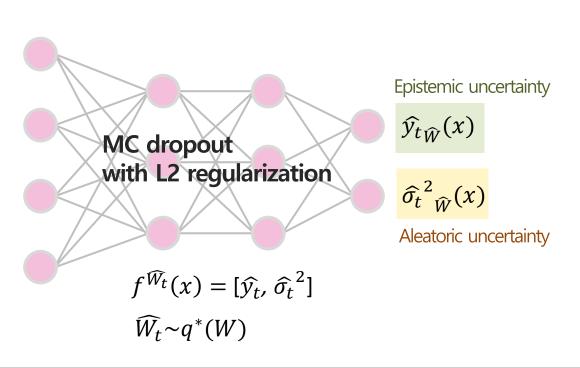


**Bayesian Neural Networks for Computer Vision** 

### Density Network Architecture

- Loss attenuation: heteroscedastic uncertainty 반영하여 더욱 강건한 모델 구축
- 불확실성이 큰 입력에 대해서는 loss에 영향을 적게 반영하기 위한 가중치를 적용





After T stochastic forward passes

$$E(y^*) \approx \frac{1}{T} \sum_{t=1}^{T} \hat{y}_t$$
 Prediction

$$Var(y^*) \approx \frac{1}{T} \sum_{t=1}^{T} \hat{y}_t^2 - \left(\frac{1}{T} \sum_{t=1}^{T} \hat{y}_t\right)^2 + \frac{1}{T} \sum_{t=1}^{T} \hat{\sigma}_t^2$$

Total uncertainty

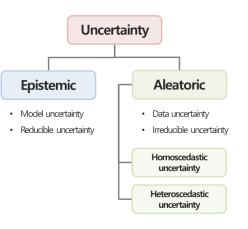
Epistemic uncertainty

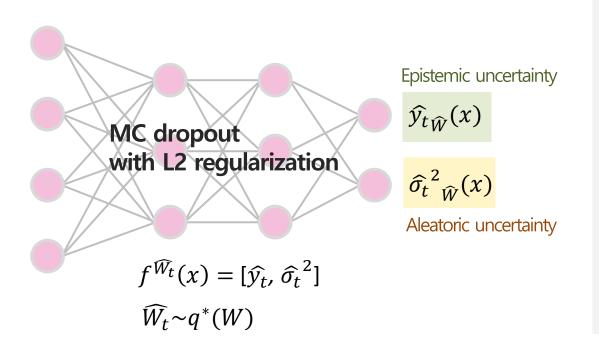
Aleatoric uncertainty

**Bayesian Neural Networks for Computer Vision** 

#### Density Network Architecture

- Loss attenuation: heteroscedastic uncertainty 반영하여 더욱 강건한 모델 구축
- 불확실성이 큰 입력에 대해서는 loss에 영향을 적게 반영하기 위한 가중치를 적용





Heteroscedastic uncertainty as learned loss attenuation

$$L_{BNN}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2\sigma(x_i)^2} \|y_i - f(x_i)\|^2 + \frac{1}{2} \log \sigma(x_i)^2$$

Residual's weight

Uncertainty regularization

Residual's weight

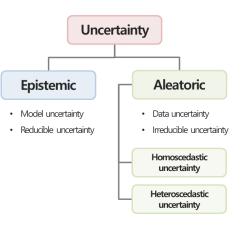
Aleatoric heteroscedastic uncertainty가 큰 예측 값에 대해서는 loss(residual)를 적게 반영

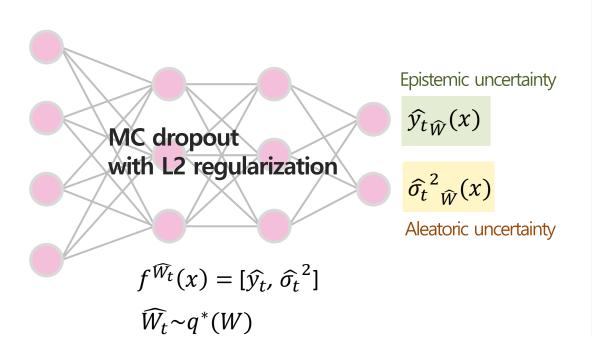
• Uncertainty regularization
Aleatoric uncertainty가 모든 데이터에 대해 무한히 커지는 것을 제약

**Bayesian Neural Networks for Computer Vision** 

#### Density Network Architecture

- Loss attenuation: heteroscedastic uncertainty 반영하여 더욱 강건한 모델 구축
- 불확실성이 큰 입력에 대해서는 loss에 영향을 적게 반영하기 위한 가중치를 적용





Heteroscedastic uncertainty as learned loss attenuation

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Residual's weight

Uncertainty regularization

• Residual's weight

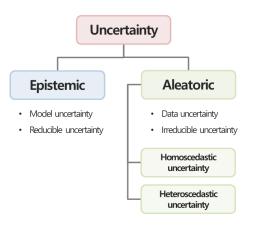
Aleatoric heteroscedastic uncertainty가 큰 예측 값에 대해서는 loss(residual)를 적게 반영

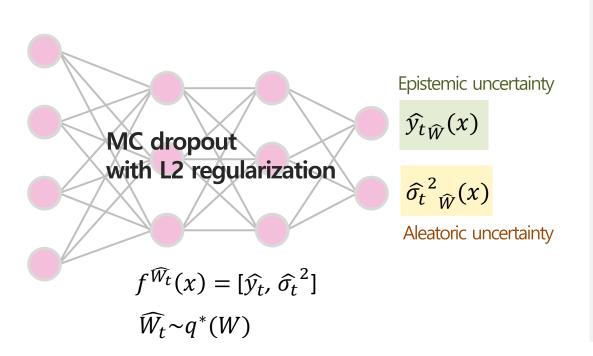
• 노이즈가 큰 데이터(높은 heteroscedastic uncertainty가 예측된 값)에 대해서는 loss에 적게 반영

**Bayesian Neural Networks for Computer Vision** 

#### Density Network Architecture

- Loss attenuation: heteroscedastic uncertainty 반영하여 더욱 강건한 모델 구축
- 불확실성이 큰 입력에 대해서는 loss에 영향을 적게 반영하기 위한 가중치를 적용





Heteroscedastic uncertainty as learned loss attenuation

$$L_{BNN}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2\sigma(x_i)^2} \|y_i - f(x_i)\|^2 + \frac{1}{2} \log\sigma(x_i)^2$$

Residual's weight

Uncertainty regularization

#### • Residual's weight

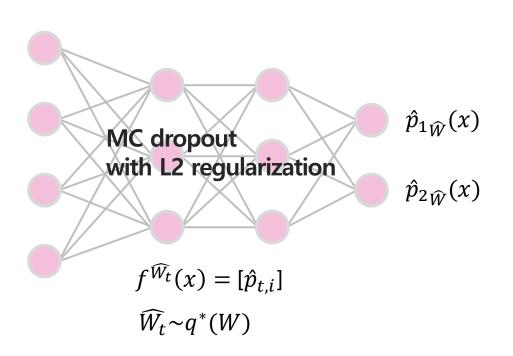
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**Bayesian Neural Networks for Computer Vision** 

#### Density Network Architecture for classification

- 후속 연구들에 classification에 적합하도록 수식을 정리하고자 하는 시도가 있음
- $\hat{\sigma}^2 = \hat{p}(1-\hat{p}) = \hat{p} \hat{p}^2$  으로 도출할 수 있음을 활용하여 별도의 node로 구분하지 않음 Aleatoric uncertainty



After T stochastic forward passes

$$Var(y^*) \approx \frac{1}{T} \sum_{t=1}^{T} \hat{y}_t^2 - \left(\frac{1}{T} \sum_{t=1}^{T} \hat{y}_t\right)^2 + \frac{1}{T} \sum_{t=1}^{T} \hat{\sigma}_t^2$$

Total uncertainty

Epistemic uncertainty

Aleatoric uncertainty

$$Var(y^*) \approx \frac{1}{T} \sum_{t=1}^{T} \hat{p}_{t,i}^2 - \left(\frac{1}{T} \sum_{t=1}^{T} \hat{p}_{t,i}\right)^2 + \frac{1}{T} \sum_{t=1}^{T} \hat{p}_{t,i} - \hat{p}_{t,i}^2$$

Total uncertainty

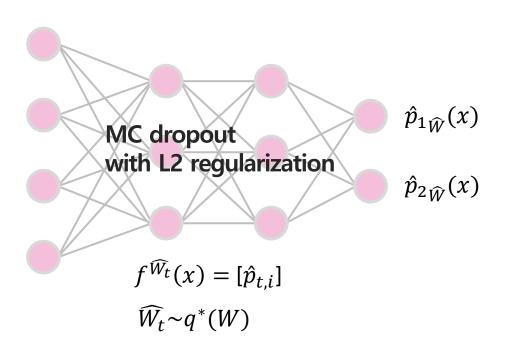
Epistemic uncertainty

Aleatoric uncertainty

**Bayesian Neural Networks for Computer Vision** 

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Total uncertainty

Epistemic uncertainty

Aleatoric uncertainty

$$Var(y^*) \approx \frac{1}{T} \sum_{t=1}^{T} (\hat{p}_{t,i} - \bar{p}_{t,i})^2 + \frac{1}{T} \sum_{t=1}^{T} \hat{p}_{t,i} - \hat{p}_{t,i}^2$$

Total uncertainty

Epistemic uncertainty

Aleatoric uncertainty

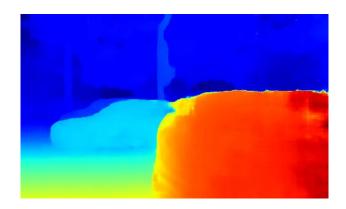
**Bayesian Neural Networks for Computer Vision Results** 

### Computer vision tasks

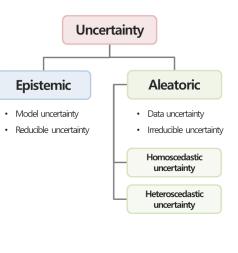
- Depth regression (regression task)
- Semantic segmentation (classification task)

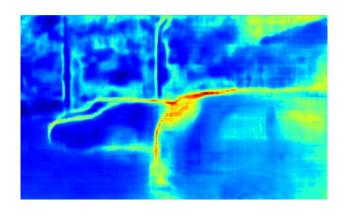


Original image



Depth regression



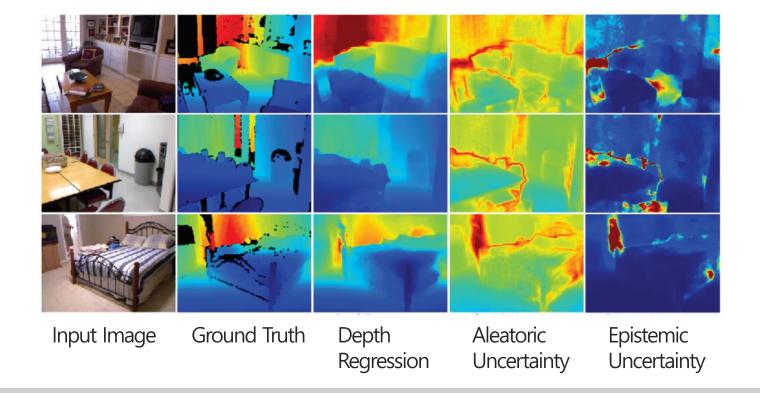


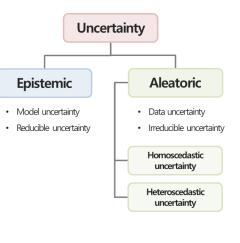
Semantic segmentation

**Bayesian Neural Networks for Computer Vision Results** 

### Depth regression

- 테두리에 대한 예측에 high aleatoric uncertainty
- 예측이 틀린 부분에 high epistemic uncertainty





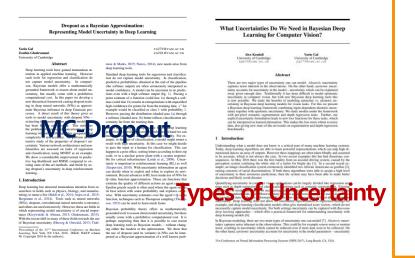
# Uncertainty

# Quantification

예측에 대한 불확실성

잘 정량화하자

### **Bayesian-based Approach**



## **Ensemble-based Approach**



### **GP-based Approach**





Simple and Scalable Deep Ensembles

#### ❖ NeurlPS 2017 (22년 1월 기준 2257건 인용)

- 기존의 BNN의 경우, 모델 구조가 한정적, 계산량多
- Ensemble을 이용하여 간단하게 uncertainty 모델링

#### Simple and Scalable Predictive Uncertainty Estimation using Deep Ensembles

Balaji Lakshminarayanan Alexander Pritzel Charles Blundell DeepMind

{balajiln,apritzel,cblundell}@google.com

#### Abstract

Deep neural networks (NNs) are powerful black box predictors that have recently achieved impressive performance on a wide spectrum of tasks. Quantifying predictive uncertainty in NNs is a challenging and yet unsolved problem. Bayesian NNs, which learn a distribution once weights, are currently the state-of-the-art for estimating predictive uncertainty; however these require significant modifications to the training procedure and are computationally expensive compared to into the training procedure and are computationally expensive compared to its simple to implement, readily parallelizable, requires very little hyperparameter tuning, and yelds high quality predictive uncertainty estimates. Through a series of experiments on classification and regression benchmarks, we demonstrate that our method produces well-calibrated uncertainty estimates which are as good or better than approximate Bayesian NNs. To assess robustness to dataset shift, we evaluate the predictive uncertainty on test examples from known and unknown distributions, and show that our method of positive to examples. We demonstrate that exabibility of our method by out-of-distribution examples. We demonstrate the scalability of our method by

#### 1 Introduction

Deep neural networks (NNs), have achieved date-of-the-art performance on a wide variety of muchine tearning tasks [35] and are becoming increasingly popular in domains such as computer vision [32], speech recognition [25], natural language processing [42], and bioinformatics [2, 61]. Despite impressive accuracies in supervised learning benchmarks. NNs are poor at quantifying predictive uncertainty, and tend to produce overconfident predictions. Overconfident incorrect predictions can be harmful or offensive [3], hence proper uncertainty quantification is crucial for practical applications.

Evaluating the quality of predictive uncertainties is challenging as the 'ground truth' uncertainty estimates are usually not available. In this work, we shall focus upon two-valuation measures that are motivated by practical applications of NNs. Firstly, we shall examine calibration [12, 13], a frequentist notion of uncertainty which measures the discrepancy between subjective forecasts and compriscal long-run frequencies. The quality of calibration can be measured by proper scoring rules [17] such as long predictive probabilities and the Brier score [9]. Note that calibration is an orthogonal concern to accuracy: a network's predictions may be accurate and yet miscalibrated, and vice versa. The second notion of quality of predictive uncertainty we consider concerns generalization of the predictive uncertainty to dromain shift (also referred to as out-of-distribution examples [23]), that its reasons of the network knows what it knows. For example, if a network trained on one dataset is evaluated on a completely different dataset, then the network should output high predictive uncertainty as inputs from a different dataset unuel be far away from the training data. Well-calibrated predictions that are robust to model diagnosis).

31st Conference on Neural Information Processing Systems (NIPS 2017), Long Beach, CA, USA

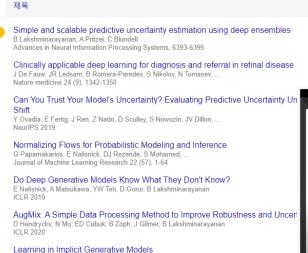


S Mohamed, B Lakshminaravanan

arXiv preprint arXiv:1610.03483

Balaji Lakshminarayanan Staff Research Scientist at <u>Google</u> Brain google.com의 이메일 확인됨 - <u>홈페이지</u> Machine Learning

http://www.gatsby.ucl.ac.uk/~balaji/



Uncertainty in Deep Learning

Balaji Lakshminarayanan, Dustin Tran, Jasper Snoek

Week 5-Uncertainty and Out-of-Distribution Robustness in Deep Learning

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Simple and Scalable Predictive Uncertainty

Estimation using Deep Ensembles

Balaji Lakshminarayanan Alexander Pritzel Charles Blundell DeepMind {balajiln,apritzel,cblundell}@google.com

Abstract

Deep neural networks (NNs) are powerful black box predictors that have recently achieved impressive performance on a wide spectrum of tasks. Quantifying predictive uncertainty in NNs is a challenging and yet unsolved problem. Bayesian

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31st Conference on Neural Information Processing Systems (NIPS 2017), Long Beach, CA, USA

1 Introduction

(e.g., weather forecasting, medical diagnosis).

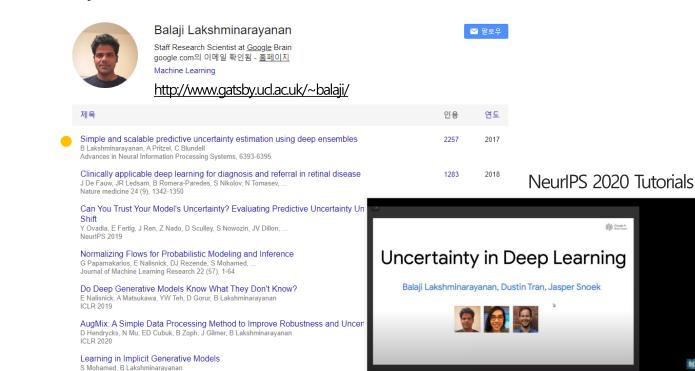
Simple and Scalable Deep Ensembles

#### ❖ NeurlPS 2017 (22년 1월 기준 2257건 인용)

• 기존의 BNN의 경우, 모델 구조가 한정적, 계산량多 → 모델 구조의 제약 없애서 Simple

arXiv preprint arXiv:1610.03483

• Ensemble을 이용하여 간단하게 uncertainty 모델링 → 병렬연산을 수행하여 Scalable



Week 5 - Uncertainty and Out-of-Distribution Robustness in Deep Learning



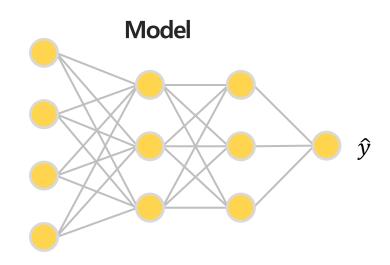
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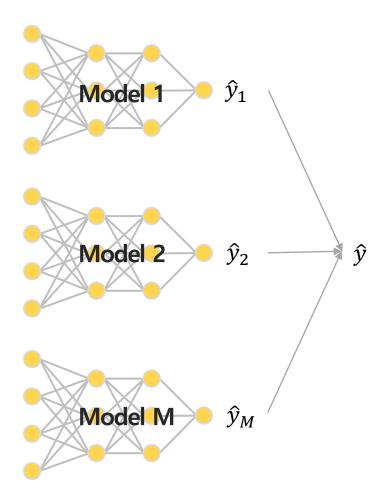
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Simple and Scalable Deep Ensembles

#### Ensemble

- 다수의 결과를 종합하여 최종 예측을 수행
- 대표적으로 배깅(Bagging)과 부스팅(Boosting)으로 구분

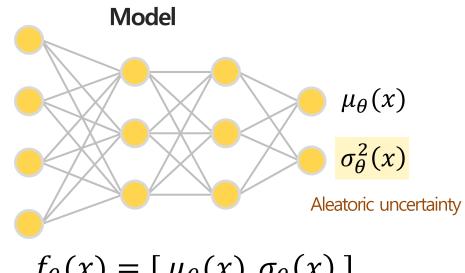


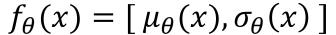


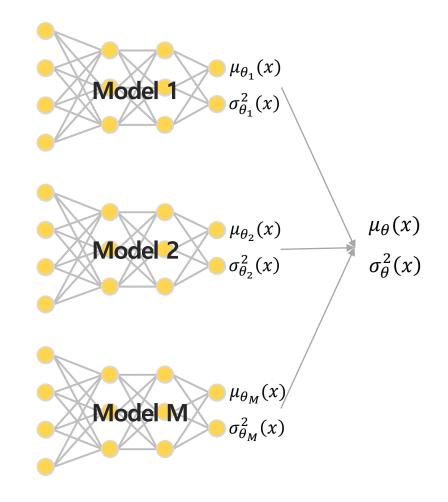
Simple and Scalable Deep Ensembles

#### **❖** Ensemble + Uncertainty (Aleatoric) = Deep Ensembles

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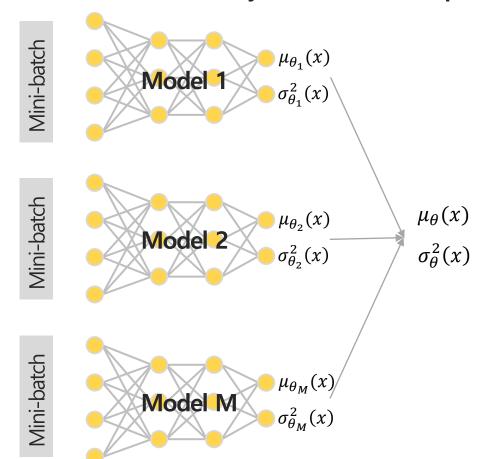






Simple and Scalable Deep Ensembles

**❖** Ensemble + Uncertainty (Aleatoric) = Deep Ensembles



M 개의 mini-batch 구성하여 학습 수행 후,

$$\mu_{\theta}(x) = \frac{1}{M} \sum_{m=1}^{M} \mu_{\theta_m}(x)$$
 Prediction

$$\sigma_{\theta}^{2}(x) = \frac{1}{M} \sum_{m=1}^{M} (\sigma_{\theta_{m}}^{2}(x) + \mu_{\theta_{m}}^{2}(x)) - \mu_{\theta}^{2}(x)$$

Uncertainty (Aleatoric uncertainty)

Simple and Scalable Deep Ensembles

#### Proper scoring rules

- 모델의 학습 기준으로 scoring rule을 활용
- 일반적인 loss function (cross entropy, Brier score)은 scoring rule을 만족
- Regression에서는 불확실성 정량화가 가능한 새로운 scoring rule을 제안

Heteroscedastic uncertainty as learned loss attenuation

$$L_{BNN}(\theta) = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{2\sigma(x_i)^2} \|y_i^{\text{MSE}} - f(x_i)\|^2 + \frac{1}{2} \log \sigma(x_i)^2$$

Residual's weight

Uncertainty regularization

· Residual's weight

Aleatoric heteroscedastic uncertainty가 큰 예측 값에 대해서는 loss(residual)를 적게 반영

· Uncertainty regularization

Aleatoric uncertainty가 모든 데이터에 대해 무한히 커지는 것을 제약

#### Classification

Brier score: 실제 label의 one-hot 벡터와 예측확률 사이의
 MSE (mean squared error)

$$L_{Ensemble}(\theta) = \frac{1}{C} \sum_{c=1}^{C} \left( \delta_{c=y} - p_{\theta}(y=c|x) \right)^{2}$$

 $\delta_{c=y}$ : 실제 label의 one-hot encoding 벡터 [1.0,0.0]

 $p_{\theta}(y = c|x)$ :예측 확률 [0.8, 0.2]

#### Regression

- $\sigma_{\theta}^2(x)$ 을 반영하여 MSE 보정
- Negative Log-likelihood(NLL)

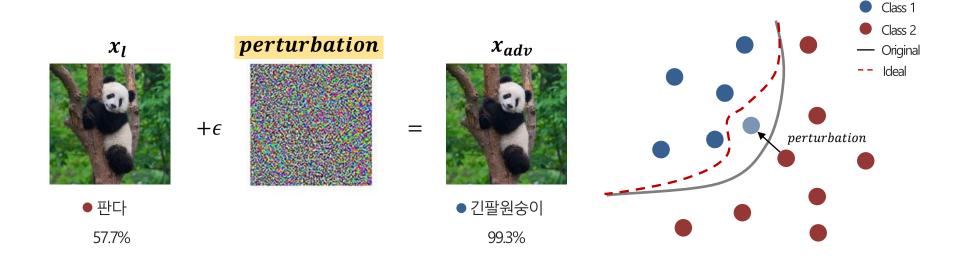
$$L_{Ensemble}(\theta) = \frac{\left(y - \mu_{\theta}(x)\right)^{2}}{2\sigma_{\theta}^{2}(x)} + \frac{\log \sigma_{\theta}^{2}(x)}{2} + constant$$

Residual's weight

Simple and Scalable Deep Ensembles

#### Adversarial Training

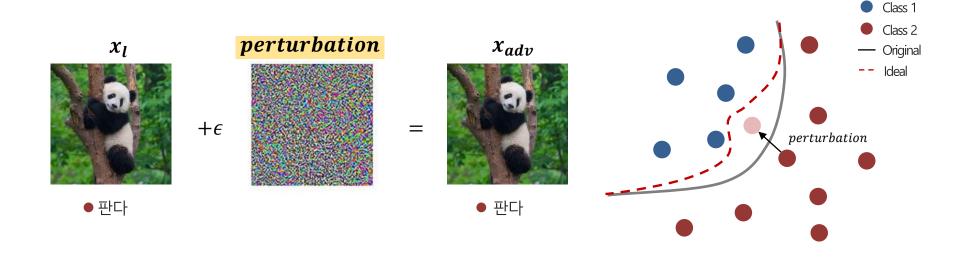
- Adversarial training은 adversarial example을 포함하여 모델의 과적합을 방지하는 기법
- Adversarial example은 일종의 augmentation기법
- 사람의 눈에는 동일해 보이지만, 모델은 <mark>헷갈려 하는 데이터를 perturbation을 더함</mark>으로써 생성함 손실(loss)을 최대화 하는 방향



Simple and Scalable Deep Ensembles

#### Adversarial Training

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#### Simple and Scalable Deep Ensembles

#### ❖ Deep Ensemble Training Procedure

#### Algorithm 1 Pseudocode of the training procedure for our method

- 1:  $\triangleright$  Let each neural network parametrize a distribution over the outputs, i.e.  $p_{\theta}(y|\mathbf{x})$ . Use a proper scoring rule as the training criterion  $\ell(\theta, \mathbf{x}, y)$ . Recommended default values are M = 5 and  $\epsilon = 1\%$  of the input range of the corresponding dimension (e.g 2.55 if input range is [0,255]).
- 2: Initialize  $\theta_1, \theta_2, \dots, \theta_M$  randomly
- 3: **for** m = 1 : M **do**

- ⊳ train networks independently in parallel
- 4: Sample data point  $n_m$  randomly for each net  $\triangleright$  single  $n_m$  for clarity, minibatch in practice
- 5: Generate adversarial example using  $\mathbf{x}'_{n_m} = \mathbf{x}_{n_m} + \epsilon \operatorname{sign}(\nabla_{\mathbf{x}_{n_m}} \ell(\theta_m, \mathbf{x}_{n_m}, y_{n_m}))$
- 6: Minimize  $\ell(\theta_m, \mathbf{x}_{n_m}, y_{n_m}) + \ell(\theta_m, \mathbf{x}'_{n_m}, y_{n_m})$  w.r.t.  $\theta_m > adversarial training (optional)$
- 1. Loss function, 네트워크 개수 M, adversarial training ratio  $\epsilon$  정의
- 2. 각 네트워크의 파라미터 초기화
- 3. M개의 네트워크에 대해 반복 수행 (독립적으로 병렬처리 가능)
  - 4. 전체 데이터 셋에서 각 네트워크를 학습시키기 위한 mini-batch 데이터셋 구축
  - 5. 해당 mini-batch에 대한 adversarial example 생성하여 데이터 증폭 (optional)
  - 6. Score rule인 loss를 최소화 하도록 네트워크 파라미터 학습

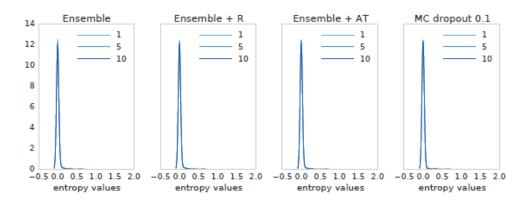
- Deep ensembles
- Adversarial training
- Score rule



Simple and Scalable Deep Ensembles Results

- ❖ 학습에 활용하지 않은 Out-of-distribution데이터에 대한 불확실성 검증
  - 분류 문제상황에서 불확실성에 대한 지표로 entropy를 확인
  - R: random noise / AT: adversarial training
  - MC dropout (첫번째 논문과 비교)

Train: MNIST Test: MNIST



#### **MNIST**



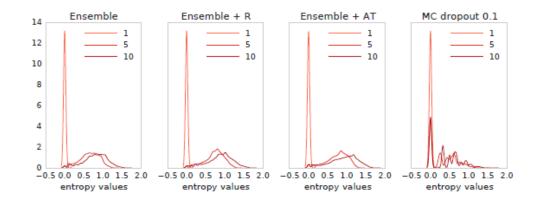
**Not-MNIST** 



#### **Out-of-distribution**

Train: MNIST

Test: Not-MNIST



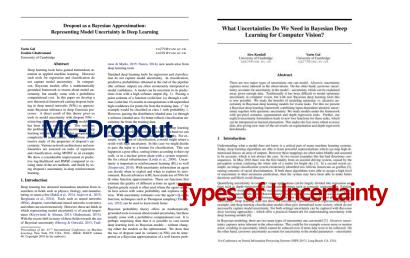
# Uncertainty

# Quantification

예측에 대한 불확실성

잘 정량화하자

### **Bayesian-based Approach**



## **Ensemble-based Approach**



## **GP-based Approach**





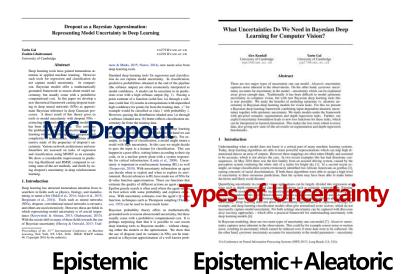
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**Ensemble-based Approach** 



**GP-based Approach** 





#### **Spectral-normalized Neural Gaussian Process**

- [1] Liu, Jeremiah Zhe, et al. "Simple and principled uncertainty estimation with deterministic deep learning via distance awareness." arXiv preprint arXiv:2006.10108 (2020).
- [2] Abdar, Moloud, et al. "A review of uncertainty quantification in deep learning: Techniques, applications and challenges." *Information Fusion* (2021).
- [3] Nado, Zachary, et al. "Uncertainty Baselines: Benchmarks for Uncertainty & Robustness in Deep Learning." *arXiv* preprint arXiv:2106.04015 (2021).

### ❖ NeurlPS 2020 Tutorials (22년 1월 기준 68건 인용)

- 최근 Uncertainty quantification 리뷰 문헌[2], Benchmark 정리 문헌[3] 에서 SNGP가 주로 언급
- 거리를 반영하여 Gaussian process를 설계한 방법론

#### Simple and Scalable Predictive Uncertainty Estimation using Deep Ensembles

Balaji Lakshminarayanan Alexander Pritzel Charles Blundell DeepMind {balajiln,apritzel,cblundell}@google.com

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#### Introduction

Deep neural networks (NNs) have achieved state-of-the-art performance on a wide variety of machine learning tasks [35] and are becoming increasingly oppular in domains such as computer vision [32], speech recognition [25], natural language processing [42], and bioinformatics [2, 61]. Despite impressive accuracies in supervised learning benchmarks, NNs are poor at quantityin predictions can be a produced by the produce overconfident predictions. Overconfident incorrect predictions can barmful or offensive [3], hence proper uncertainty quantification is crucial for practical applications.

Evaluating the quality of predictive uncertainties is challenging as the 'ground truth' uncertainty estimates are usually not available. In this work, we shall focus upon two evaluation measures that are motivated by practical applications of NNs. Firstly, we shall examine cultivation [12, 13], a frequentist notion of uncertainty which measures the discrepancy between subjective forecasts and (empirical) long-run frequencies. The quality of calibration can be measured by proper scoring rules of the contraction of contractions are more predictive probabilities and the Brite score [9]. Note that calibration is an orthogonal experiment of contractions are not predictive probabilities and the first resort of predictive uncertainty to domain shift falso referred to as one-of-distribution examples [23], that is, measuring if the network knows what it knows. For example, if a network trained on one dataset is evaluated on a completely different dataset, then the network should output high predictive uncertainty as inputs from a different dataset would be far away from the training data. Well-calibrated predictions that are robust to model missage-discretion and dataset shift have a number of important practical uses

31st Conference on Neural Information Processing Systems (NIPS 2017), Long Beach, CA, USA

#### Simple and Principled Uncertainty Estimation with Deterministic Deep Learning via Distance Awareness

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#### Abstract

Bayesian neural networks and deep ensembles are principled approaches to estimate the predictive uncertainty of a deep learning model. However their practicality in real-time, industrial-scale applications are limited due to their heavy memory and inference cost. This motivates us to study principled approaches to high-quality uncertainty estimation that require only a single deep neural network (INNN). By formalizing the uncertainty estimation as a minimax learning problem, we first tance of a testing example from the training data manifold, as a necessary condition for a DNN to achieve high-quality (e., minimax optimal) uncertainty estimation. We then propose Spectral-normalized Neural Caussian Process (SNCP), a single a weight normalization step during training and replacing the output layer with a Caussian Process. On a soil of vision and language understanding tasks and on Caussian Process. On a soil of vision and language understanding tasks and on ensembles in prediction, calibration and out-of-domain detection, and outperforms the other single-model approaches?

#### 1 Introduction

Efficient methods that reliably quantify a deep neural network (DNN)'s predictive uncertainty are important for industrial-scale, real-word applications, which include examples such as object recognition in autonomous driving [22], ad click prediction in online advertising [76], and intent understanding in conversational system [84]. For example, for a natural language understanding (NLU) model built for a domain-specific charbot service (e.g., weather inquiry), the such s' indirection whether to abstain or to trigger one of its known APIs.

When deep classifiers make predictions on input examples that are far from the support of the training set, their performance can be arbitrarily bud [4,14]. This motivates the meed for methods that are aware of the distance between an input test example and previously seen training examples, so they can return a uniform (i.e. maximum entropy) distribution over output labels if the input is too far from the training set (i.e., the input is out-of-domain) [30]. Gaussian processes (GPN) with suitable kernels only owed a property. However, to apply Gaussian processes to alphy-dimensional machine

34th Conference on Neural Information Processing Systems (NeurIPS 2020), Vancouver, Canada



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☑ 팔로우

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Can You Trust Your Model's Uncertainty? Evaluating Predictive Uncertainty Under Dataset Shift Y Ovadia, E Fertig, J Ren, Z Nado, D Sculley, S Nowozin, JV Dillon, NeurlPS 2019	586	2019
Normalizing Flows for Probabilistic Modeling and Inference G Papamakarios, E Nalisnick, DJ Rezende, S Mohamed, Journal of Machine Learning Research 22 (57), 1-64	381	2021
Do Deep Generative Models Know What They Don't Know? E Nalisnick, A Matsukawa, YW Teh, D Gorur, B Lakshminarayanan ICLR 2019	340	2019
AugMix: A Simple Data Processing Method to Improve Robustness and Uncertainty D Hendrycks, N Mu, ED Cubuk, B Zoph, J Gilmer, B Lakshminarayanan ICLR 2020	319	2020
Learning in Implicit Generative Models S Mohamed, B Lakshminarayanan arXiv preprint arXiv:1610.03483	309	2016



<sup>\*</sup>Work done at Google Research.

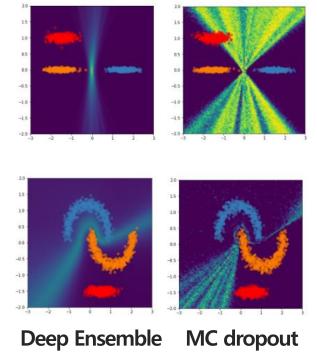
\*Work done as an Google Al Resident

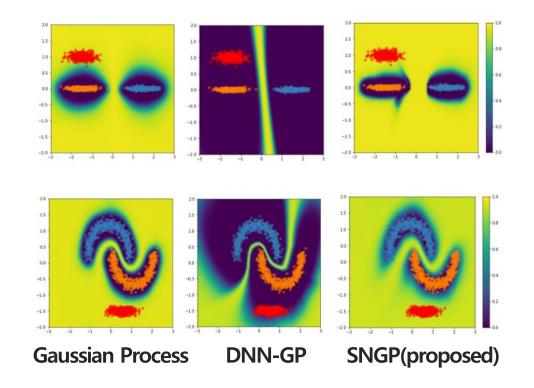
<sup>&</sup>lt;sup>3</sup>Code available at https://github.com/google/uncertainty-baselines/tree/master/baselines

Spectral-normalized Neural Gaussian Process

## ❖ 불확실성 정량화 도메인에서 Bayesian-approach, Ensemble-approach 연구들이 대표적으로 수행

- 하지만, 해당 방법론들은 out-of-distribution에 대해 적절한 불확실성 정량화가 되지 않는 한계가 있음
- Out-of-distribution: 학습에 사용한 데이터와는 '거리(차이)'가 있는 데이터
- Class 1
- Class 2
- Out-of-distribution

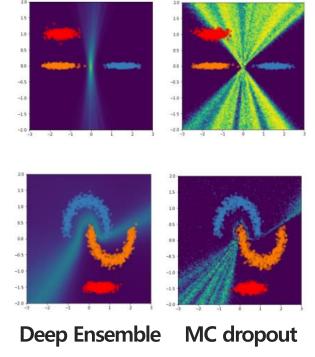


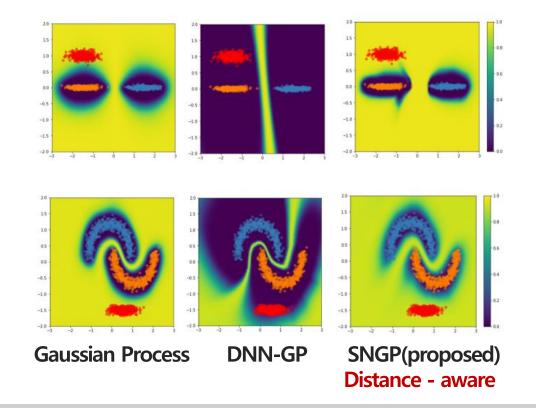


Spectral-normalized Neural Gaussian Process

## ❖ Gaussian Process를 효과적으로 구축하기 위한 접근

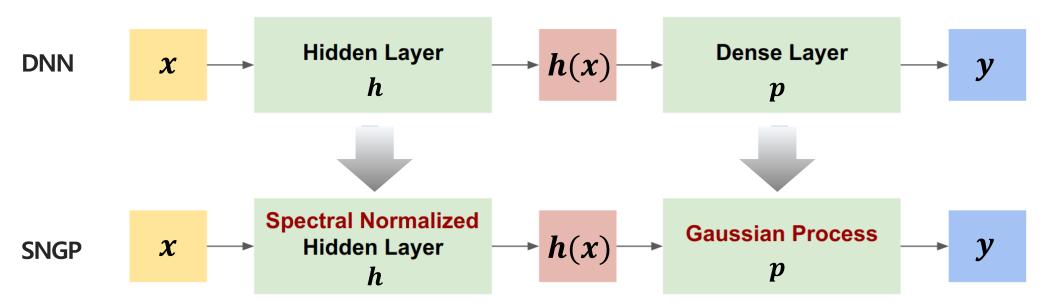
- GP는 불확실성 정량화가 적절하지만, 연산량이 매우 크므로 제약이 있으며 일반적으로는 차원축소 값이 입력에 활용
- GP layer를 활용하기 위해 '거리'정보가 유지되도록 차원축소를 적절히 수행하는 것이 중요
- Class 1
- Class 2
- Out-of-distribution





Spectral-normalized Neural Gaussian Process

- ❖ '거리'정보가 유지되도록 차원축소를 수행하여 Gaussian process layer 적용
  - GP는 불확실성 정량화가 적절하지만, 연산량이 매우 크므로 제약이 있으며 일반적으로는 차원축소 값이 입력에 활용
  - GP layer를 활용하기 위해 '거리'정보가 유지되도록 차원축소를 적절히 수행하는 것이 중요



- (1) Distance-preserving Hidden Mapping via Spectral Normalization
- 2) Distance-aware Output Layer via Laplace-approximated Neural Gaussian Process

**Spectral-normalized Neural Gaussian Process** 

## ❖ '거리'정보가 유지되도록 차원축소를 수행하여 Gaussian process layer 적용

- GP는 불확실성 정량화가 적절하지만, 연산량이 매우 크므로 제약이 있으며 일반적으로는 차원축소 값이 입력에 활용
- GP layer를 활용하기 위해 '거리'정보가 유지되도록 차원축소를 적절히 수행하는 것이 중요

#### Algorithm 1 SNGP Training

- 1: **Input:** Minibatches  $\{D_i\}_{i=1}^N$  for  $D_i = \{y_m, \mathbf{x}_m\}_{m=1}^M$ .
- 2: Initialize:

$$\hat{\Sigma} = \mathbf{I}, \mathbf{W}_L \stackrel{iid}{\sim} N(0,1), \mathbf{b}_L \stackrel{iid}{\sim} U(0,2\pi)$$

- 3: **for** train\_step = 1 **to** max\_step **do**
- 4: SGD update  $\left\{ \beta, \{\mathbf{W}_l\}_{l=1}^{L-1}, \{\mathbf{b}_l\}_{l=1}^{L-1} \right\}$
- 5: Spectral Normalization  $\{\mathbf{W}_l\}_{l=1}^{L-1}$  (10).
- 6: **if** final\_epoch **then**
- 7: Update precision matrix  $\{\hat{\Sigma}_k^{-1}\}_{k=1}^K$  (9).
- 8: **end if**
- 9: end for
- 10: Compute posterior covariance  $\hat{\Sigma}_k = inv(\hat{\Sigma}_k^{-1})$

#### Algorithm 2 SNGP Prediction

- 1: Input: Testing example x.
- 2: Compute Feature:

$$\Phi_{D_L \times 1} = \sqrt{2/D_L} * \cos(\mathbf{W}_L h(\mathbf{x}) + \mathbf{b}_L),$$

3: Compute Posterior Mean:

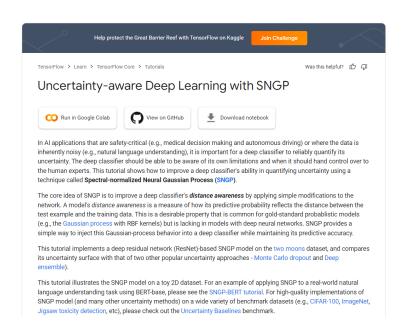
$$logit_k(\mathbf{x}) = \Phi^{\top} \boldsymbol{\beta}_k$$

4: Compute Posterior Variance:

$$\operatorname{var}_k(\mathbf{x}) = \mathbf{\Phi}^{\top} \hat{\Sigma}_k \mathbf{\Phi}.$$

5: Compute Predictive Distribution:

$$p(y|\mathbf{x}) = \int_{m \sim N(\mathsf{logit}(\mathbf{x}), \mathsf{Var}(\mathbf{x}))} \mathsf{softmax}(m)$$



Spectral-normalized Neural Gaussian Process Results

Methods	Additional Regularization	Output Layer	Ensemble Training	Multi-pass Inference
Deterministic	-	Dense	-	-
MC Dropout	Dropout	Dense	-	Yes
Deep Ensemble	-	Dense	Yes	Yes
MCD-GP	Dropout	GP	-	Yes
DUQ	Gradient Penalty	RBF	-	-
DNN-SN	Spec Norm	Dense	-	-
DNN-GP	-	GP	-	-
SNGP	Spec Norm	GP	-	-

#### ❖ CIFAR-10에 Wide ResNet구조 / CLINC OOS(intent)에 BERT구조

- Accuracy, ECE, NLL 지표에서 기존에 SOTA방법론인 Deep Ensembles와 유사한 수준의 성능 도출
- 특히, Out-of-distribution을 탐지하는 성능이 가장 우수하였으며, 연산 효율성(latency) 측면에서는 SOTA대비 월등히 개선

#### Vision task

		acy (†)	ECE		NLI		OOD A	Latency (↓)	
Method	Clean	Corrupted	Clean	Corrupted	Clean	Corrupted	SVHN	CIFAR-100	(ms / example)
Deterministic	$96.0 \pm 0.01$	$72.9 \pm 0.01 \mid$	$0.023\pm0.002$	$0.153 \pm 0.011$	$0.158 \pm 0.01$	$1.059 \pm 0.02 \mid$	$0.781 \pm 0.01$	$0.835\pm0.01$	3.91
MC Dropout	$96.0 \pm 0.01$	$70.0 \pm 0.02$	$0.021 \pm 0.002$	$0.116\pm0.009$	$0.173 \pm 0.01$	$1.152 \pm 0.01$	$0.971 \pm 0.01$	$0.832 \pm 0.01$	27.10
Deep Ensembles	$\textbf{96.6} \pm \textbf{0.01}$	$\textbf{77.9} \pm \textbf{0.01}$	$\textbf{0.010} \pm \textbf{0.001}$	$\textbf{0.087} \pm \textbf{0.004}$	$  \textbf{ 0.114} \pm \textbf{ 0.01}$	$0.815 \pm 0.01$	$0.964 \pm 0.01$	$\underline{0.888 \pm 0.01}$	38.10
MCD-GP	$95.5 \pm 0.02$	$70.0 \pm 0.01$	$0.024 \pm 0.004$	$0.100\pm0.007$	$0.172 \pm 0.01$	$1.157 \pm 0.01$	$0.960 \pm 0.01$	$0.863 \pm 0.01$	29.53
DUQ	$94.7 \pm 0.02$	$71.6 \pm 0.02$	$0.034 \pm 0.002$	$0.183 \pm 0.011$	$0.239 \pm 0.02$	$1.348 \pm 0.01$	$0.973 \pm 0.01$	$0.854 \pm 0.01$	8.68
DNN-SN	$96.0 \pm 0.01$	$72.5 \pm 0.01$	$0.025 \pm 0.004$	$0.178 \pm 0.013$	$0.171 \pm 0.01$	$1.306 \pm 0.01$	$0.974 \pm 0.01$	$0.859 \pm 0.01$	5.20
DNN-GP	$95.9 \pm 0.01$	$71.7 \pm 0.01$	$0.029 \pm 0.002$	$0.175 \pm 0.008$	$0.221 \pm 0.02$		$0.976 \pm 0.01$	$0.887 \pm 0.01$	5.58
SNGP (Ours)	$95.9 \pm 0.01$	$74.6 \pm 0.01$	$0.018 \pm 0.001$	$0.090 \pm 0.012$	$0.138 \pm 0.01$	$0.935 \pm 0.01$	$\textbf{0.990} \pm \textbf{0.01}$	$\textbf{0.905} \pm \textbf{0.01}$	6.25

#### **NLP** task

	Accuracy (†)	ECE (↓)	NLL (↓)	OC	OD	Latency (↓)
Method				AUROC (†)	AUPR (†)	(ms / example)
Deterministic	$96.5\pm0.11$	$0.024\pm0.002$	$3.559 \pm 0.11$	$0.897 \pm 0.01$	$0.757\pm0.02$	10.42
MC Dropout	$96.1 \pm 0.10$	$0.021 \pm 0.001$	$1.658 \pm 0.05$	$0.938 \pm 0.01$	$0.799 \pm 0.01$	85.62
Deep Ensemble	$\textbf{97.5} \pm \textbf{0.03}$	$\textbf{0.013} \pm \textbf{0.002}$	$\textbf{1.062} \pm \textbf{0.02}$	$\underline{0.964 \pm 0.01}$	$\underline{0.862 \pm 0.01}$	84.46
MCD-GP	$95.9 \pm 0.05$	$0.015 \pm 0.003$	$1.664 \pm 0.04$	$0.906 \pm 0.02$	$0.803 \pm 0.01$	88.38
DUQ	$96.0 \pm 0.04$	$0.059 \pm 0.002$	$4.015 \pm 0.08$	$0.917 \pm 0.01$	$0.806 \pm 0.01$	15.60
DNN-SN	$95.4 \pm 0.10$	$0.037 \pm 0.004$	$3.565 \pm 0.03$	$0.922 \pm 0.02$	$0.733 \pm 0.01$	17.36
DNN-GP	$95.9 \pm 0.07$	$0.075 \pm 0.003$	$3.594 \pm 0.02$	$0.941 \pm 0.01$	$0.831 \pm 0.01$	18.93
SNGP	$96.6 \pm 0.05$	$\underline{0.014 \pm 0.005}$	$1.218 \pm 0.03$	$\textbf{0.969} \pm \textbf{0.01}$	$\textbf{0.880} \pm \textbf{0.01}$	17.36



## **Conclusions**

**Uncertainty Quantification in Deep Learning** 

❖ 대표적인 uncertainty quantification방법론에 대해 살펴보고, 특징 파악

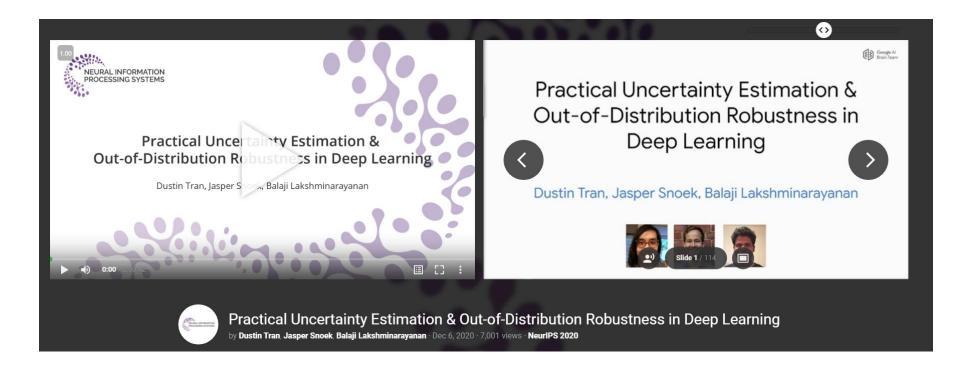


- ❖ 각 문헌마다 배경이 되는 지식을 많이 필요로 하며, 이해하는데 필수적으로 선행되어야 함
- ❖ 최신 uncertainty 연구 동향은 uncertainty 유형 세분화 보다는 'Robustness', 'Calibration', 'Out-of-Distribution' 과 같은 키워드와 연계되어 수행되고 있음 (예측 확률을 잘 정의하자)
- ❖ 최근 benchmark에 대한 프로토콜을 정의하고자 하는 시도가 있음
- ❖ 향후 다양한 application에서 적용 가능할 것이라 기대
  - Explainable AI, Medical imaging, Autonomous vehicle, Active learning, Out-of-distribution detection

## **Materials**

#### **Tutorials**

- https://neurips.cc/media/neurips-2020/Slides/16649.pdf
- https://slideslive.com/38935801/practical-uncertainty-estimation-outofdistribution-robustness-in-deep-learning
- https://www.youtube.com/watch?v=ssD7jNDIL2c

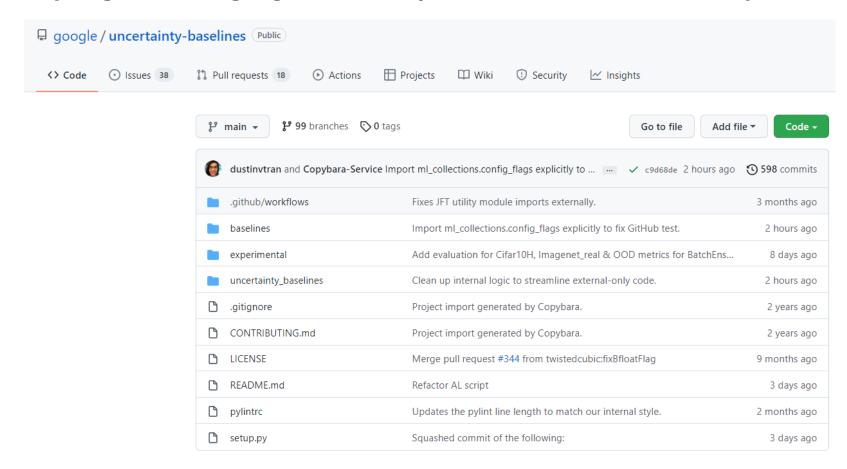




## **Materials**

#### **Github**

https://github.com/google/uncertainty-baselines Smoothness assumption



## References

#### ❖ Bayesian Neural Nets

- https://www.edwith.org/bayesiandeepleaming
- http://dmqa.korea.ac.kr/activity/seminar/252
- https://www.slideshare.net/rsilveira79/uncertainty-in-deep-learning
- https://alexgkendall.com/computer\_vision/bayesian\_deep\_learning\_for\_safe\_ai/
- https://getpocket.com/redirect?url=http%3A%2F%2Fmlg.eng.cam.acuk%2Fyarin%2Fblog\_3d801aa532c1ce.html
- https://towardsdatascience.com/building-a-bayesian-deep-learning-dassifier-ece1845bc09

#### ❖ Variational Inference

http://dmqa.korea.ac.kr/activity/seminar/253



# Thank you



# Appendix

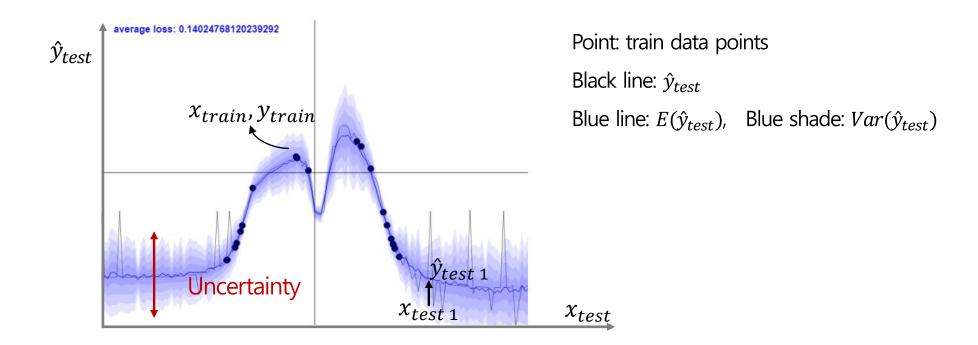


## Introduction

Uncertainty

#### Bayesian Neural Networks

- http://mlg.eng.cam.ac.uk/yarin/blog 3d801aa532c1ce.html
- Parameter w 에 분포를 가정하여, 예측값의 형태가 분포를 추정하기 때문에 구간의 형태로 도출할 수 있음
- 이때, 예측값의 분산 ≈ Confidence Interval ≈ Uncertainty

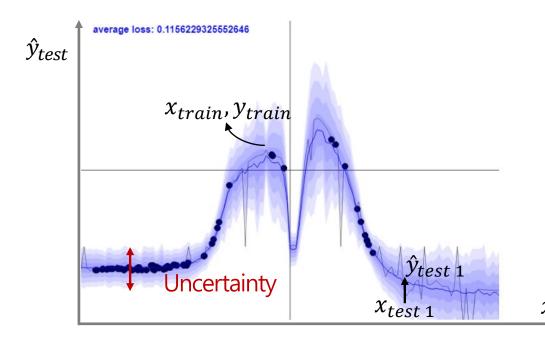


## Introduction

Uncertainty

#### Bayesian Neural Networks

- http://mlg.eng.cam.ac.uk/yarin/blog 3d801aa532c1ce.html
- Parameter w 에 분포를 가정하여, 예측값의 형태가 분포를 추정하기 때문에 구간의 형태로 도출할 수 있음
- 이때, 예측값의 분산 ≈ Confidence Interval ≈ Uncertainty



Point: train data points

Black line:  $\hat{y}_{test}$ 

Blue line:  $E(\hat{y}_{test})$ , Blue shade:  $Var(\hat{y}_{test})$ 

새로운 train data가 주어지는 경우, Uncertainty를 줄일 수 있음

 $x_{test}$ 

## How do we measure?

**Quality of uncertainty** 

- ❖ Confidence score를 정의하는 연구에서 활용되는 지표를 활용
- ❖ Calibration: 모델의 예측 확률(confidence)이 실제 정확도(accuracy)를 얼마나 반영하는지
  - Perfect calibration: 모델의 예측 확률(confidence) = 정확도(accuracy)
  - 이에 대한 정량화 지표로 expected calibration error (ECE) 가 대표적

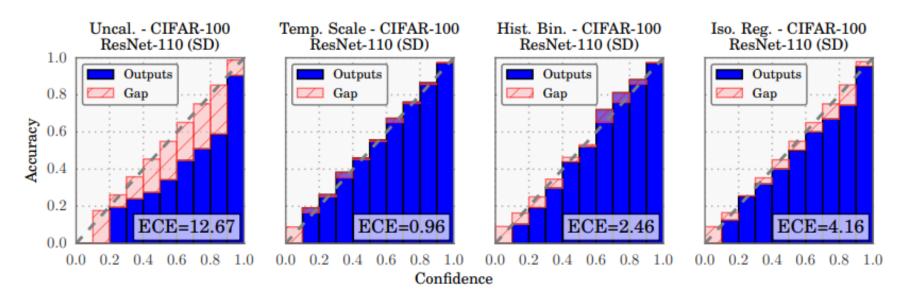


Figure 4. Reliability diagrams for CIFAR-100 before (far left) and after calibration (middle left, middle right, far right).

## How do we measure?

**Quality of uncertainty** 

- ❖ Calibration: 모델의 예측 확률(confidence)이 실제 정확도(accuracy)를 얼마나 반영하는지
- $\Leftrightarrow$  ECE=  $\sum_{b=1}^{B} \frac{n_b}{N} |acc(b) conf(b)|$ 
  - 각 bin마다 calibration error를 반영하며, 이는 정확도와는 차이가 있음

Bin 1

Model Prediction	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	Accuracy	Calibration
True Label	0	0	0	0	0	0	0	1	1	1	Bad (70%)	perfect

Bin 2

<b>Model Prediction</b>	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	Accuracy	Calibration
True Label	0	0	0	1	1	0	1	1	1	1	Bad (60%)	perfect